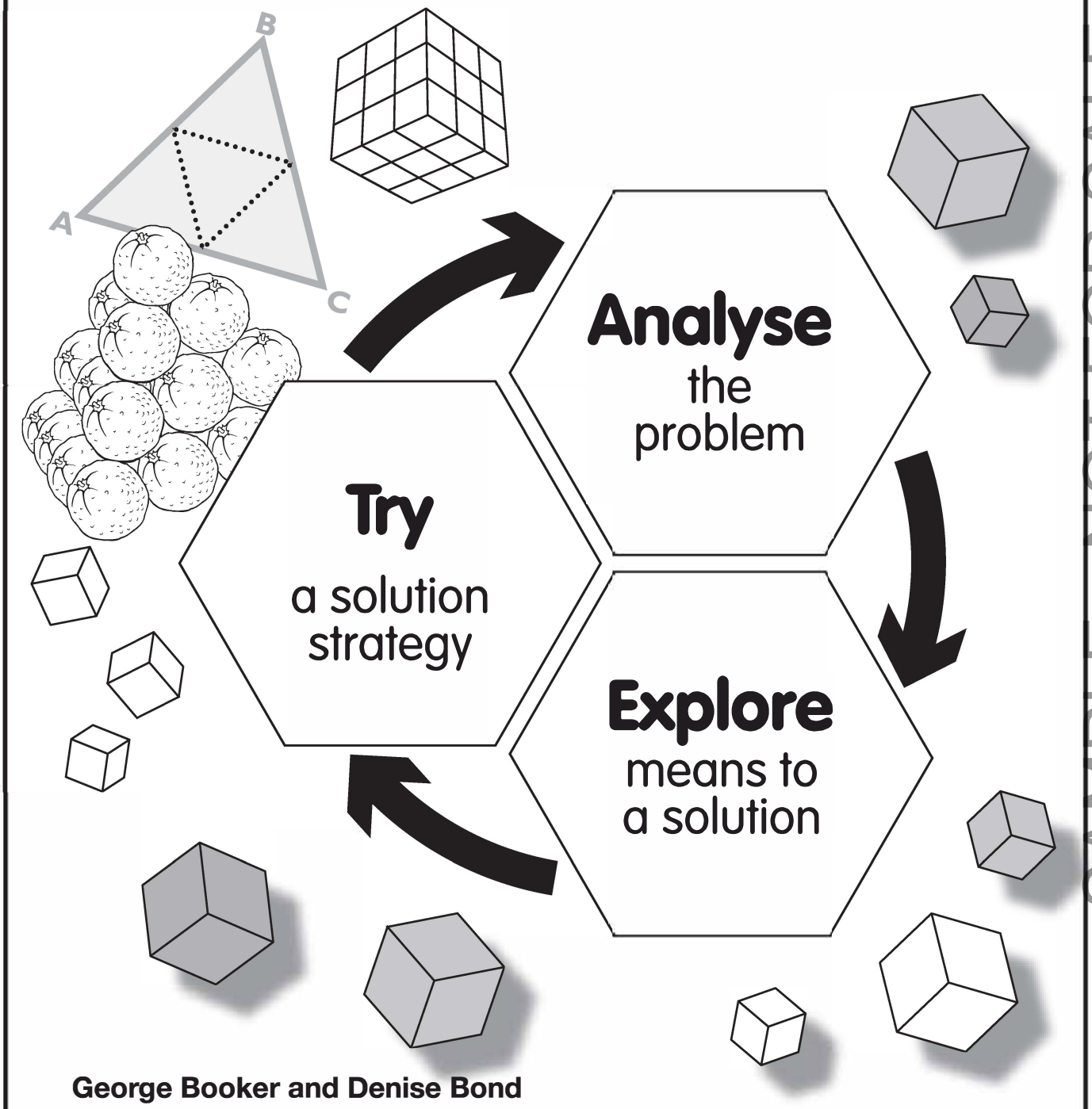
PRIMARY

BOOK D
(Ages 8–9)

CURRICULUM LINKED

Problem-solving in mathematics



George Booker and Denise Bond

SAMPLE - NOT FOR SALE

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SAMPLE - NOT FOR SALE

Problem-solving

To use spatial visualisation and measurement to solve problems.

Materials

Paper to make and fold squares and equilateral triangles, triangle and square grid paper

Focus

These pages explore ideas of perimeter by using their knowledge of squares and equilateral triangles to visualise shapes and to determine the lengths of sides within or composed of the shapes. Spatial and logical thinking, as well as numerical reasoning and organisation, are involved as students investigate the relationships among the shapes to determine the required distances.

New Zealand Curriculum links

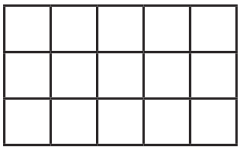
Geometry and Measurement
Measurement
• Create and use appropriate units to measure length (perimeter)

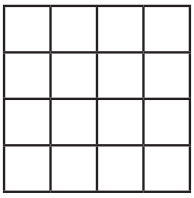
Discussion

Page 39

Students need to be able to understand that the perimeter of the first shape (made from five squares) consists of 12 sides. This can be done by counting all of the sides in a systematic way or by seeing the shape made up of symmetric parts—two sections with five sides on the top and bottom and two in the middle, for example. Since the perimeter is 36 cm, the side of each small square must be 3 cm long. It may need to be explained to students that the shape is not to scale.

This reasoning is then applied to the other shapes with the number of sides multiplied by 3 cm. For questions 2 and 3, there are several possible shapes that can be made with a perimeter of 48 cm. The only criterion is that each perimeter must use 16 sides of the smaller squares; for example, a 3-by-5 rectangle or a 4-by-4 square:

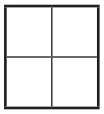




Page 40

Students need to be able to visualise the way in which the squares or triangle are folded and how the perimeter of the final shape would relate to the whole shape if unfolded. Students may need to use paper cut into squares:

Folding the square in Problem 1 in half and then half again shows 4 small squares when unfolded.

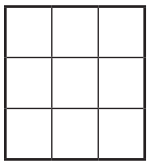


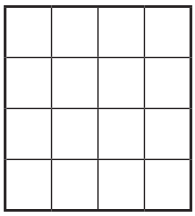
Each triangle contains 2 small squares.

The side of the rectangle, 18 cm, is the same as 6 sides of the small square, so the side of the small square is 3 cm.

The perimeter of the large square is 24 cm.

Folding the square in Problem 2 to show 9 small squares and the square in Problem 3 to show 16 small squares allows them to solved in the same way.





Page 41

This page continues the students' investigation of perimeters. While the first problem can be solved by thinking of the rectangle as being made up of two whole (9 cm) sides and two half (4.5 cm) sides, this does not readily generalise to the other ways the square is partitioned. Thinking of it as six half-sides allows the next shapes to be seen as, respectively, eight third-sides and 10 fourth-sides to readily solve the perimeter of each of the original squares. Note that thinking of these as fraction symbols ($^2/_3$ and $^2/_4$) makes the solution considerably more difficult to determine as it requires working with fractions rather than whole numbers. Another way to visualise solutions is to divide the squares and rectangles into smaller squares and see the results directly, as on the preceding pages.

Possible difficulties

- Uncertain of definition of perimeter
- Does not understand that the sides in a square or equilateral triangle are of equal length
- Unable to visualise the sides of the smaller shapes within the large shapes
- Can not keep track of the number of sides that need to be used

Extension

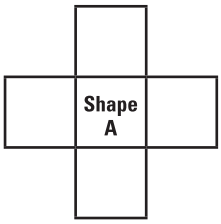
- Have students investigate shapes made from small equilateral triangles in the same way as those made from small squares.
- Ask students to create their own examples of perimeters in squares where the square is folded into 5, 6 or more rectangles.

SAMPLE - NOT FOR SALE

SQUARES AND PERIMETERS

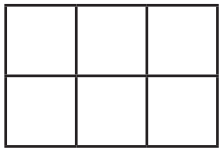
Perimeter is the distance around the boundary of a shape.

Small squares, all of the same size, have been used to make these shapes.



1. Shape A has a perimeter of 36 cm, what is the perimeter of each of these shapes?

(a)



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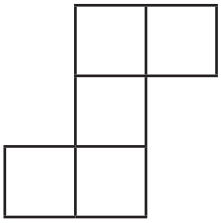
(b)



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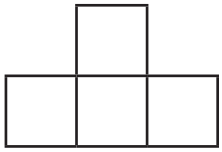
(c)



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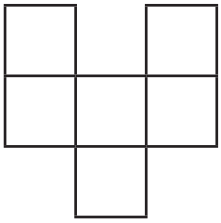
(d)



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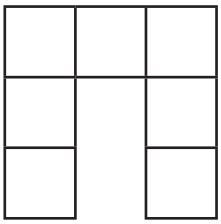
(e)



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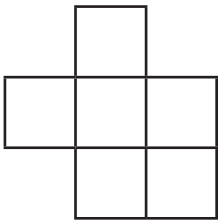
(f)



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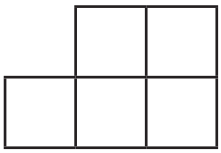
(g)



_____

2. Can you draw a shape with a perimeter of 48 cm?

3. How many different shapes with a perimeter of 48 cm can you make? Draw them.

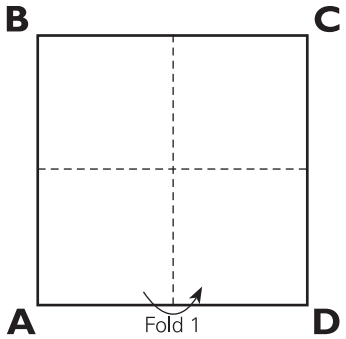
SAMPLE - NOT FOR SALE

MORE PERIMETERS

ABCD is a square sheet of paper with a perimeter of 16 cm.

Imagine that you fold the paper so that corner A folds onto corner D and corner B folds onto corner C.

Now imagine that you fold it again so that A folds onto C.

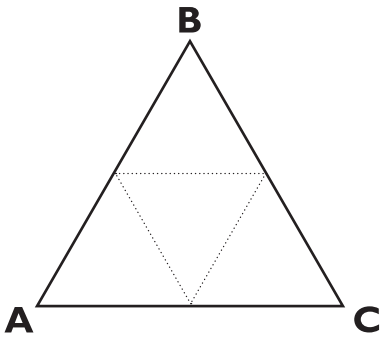


1. (a) After folding, what new shape would be created?

_____

(b) How long would its perimeter be? _____

ABC is a triangle.



2. If it were folded along the dotted lines as shown, what would the new shape be?

_____

The perimeter of the new shape is 9 cm.

3. What would the perimeter of the original triangle (ABC) have been?

_____

4. When a larger square is folded in the same way as square ABCD, the resulting shape has a perimeter of 24 cm. What is the perimeter of the original square?

_____

SAMPLE - NOT FOR SALE

PERIMETERS IN SQUARES

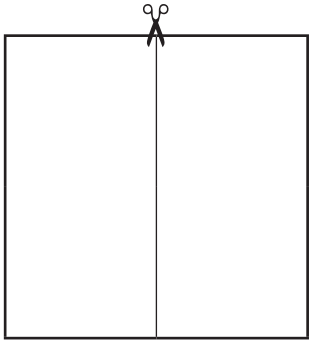
Perimeter is the distance around the boundary of a shape.

1. A small sheet of paper has been folded in half and then cut along the fold to make two rectangles.

   The perimeter of each rectangle is 18 cm.

   What was the perimeter of the original square sheet of paper?

   _____

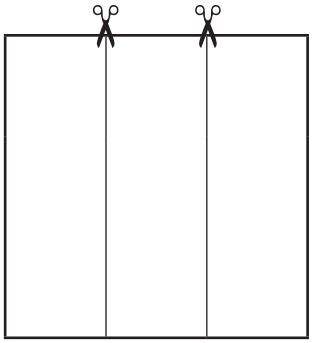


2. A larger sheet of paper has been folded into thirds and then cut along each fold to make three rectangles.

   The perimeter of each rectangle is 48 cm.

   What was the perimeter of the original square sheet of paper?

   _____

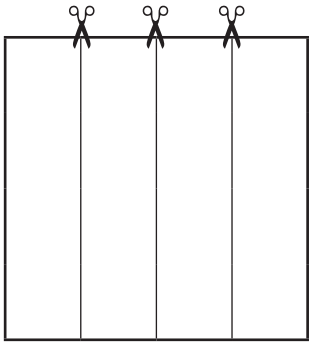


3. A large square of plastic has been folded into fourths and then cut along each fold to make four rectangles.

   The perimeter of each rectangle is 50 m.

   What was the perimeter of the original square of plastic?

   _____



SAMPLE - NOT FOR SALE

Problem-solving
To analyse and use information in word problems.

Materials
paper to fold

Focus
These pages explore word problems that require addition, subtraction, multiplication and an understanding of place value. The wording is more complex than in the previous problems that involved a combination of the three operations. Students need to determine what the problem is asking and, in many cases, carry out more than one equation in order to find solutions. Materials can be used to assist with the calculation if necessary, as these problems are about reading for information and determining what the problem is asking rather than computation or basic facts.

New Zealand Curriculum links

Number and Algebra
Number strategies
• Use simple additive strategies with whole numbers
Equations and expressions
• Communicate and explain counting, and grouping using words, pictures and numbers

Discussion
Page 43
The problems mostly require multiplication, with some addition and subtraction. Each problem is fairly straightforward, with all the information needed to find a solution readily available. The numbers have been kept simple to assist with the problem-solving process and there is no additional information that is not needed.

Students may find it helpful to draw a diagram in order to work out what is happening in the problem and to determine what needs to be multiplied to provide? A solution. Addition can be used at times rather than multiplication.

Page 44
This investigation provides information about a library, with a number of interrelated questions arising from it. The questions begin with a set number of books, magazines and CD-ROMs. Using this information as a basis, the numbers of each item are changed to meet new criteria, with some books, magazines and CD-ROMs being borrowed, returned, shelved and dusted. Students are required to keep track of the new information and use it to answer the subsequent questions.

Computation is not always needed to find a solution; for example, Problem 1 states that each shelf holds 100 books and, as there are 9472 books in the library, 95 shelves are needed. No division is necessary as place value tells us that 9472 has 94 hundreds, so 95 shelves must be needed. Similarly with Problem 2, 48 shelves are being dusted, which means 48 hundreds. Therefore, 4800 books have been dusted.

Page 45
A careful reading of each problem is needed to determine what the problem is asking. These problems focus around trains, carriages, people and seats. In some problems people are getting on and getting off and in others it is necessary to determine how many people are sitting or standing in a carriage. Each problem requires more than one step and two or more operations are needed to find a solution. There are a number of ways to find solutions and students should be encouraged to explore and try different possibilities of arriving at solutions.

Possible difficulties
- Inability to identify the need to add, subtract or multiply
- Not using place value concepts to solve the problems
- Confusion over the need to carry out more than one step to arrive at a solution
- Not understanding the concept of 'capacity'

Extension
- Students could write their own problems and give them to other students to solve.

SAMPLE - NOT FOR SALE

BEADING

1. Judy has 6 bags of coloured beads, with 42 beads in each bag. If she uses 29 beads to make a necklace, how many beads does she have left?

_____

2. Manu has 7 bags of beads. He buys 2 more bags and uses 25 beads to make some bracelets. If each bag contains 32 beads, how many beads does he have now?

_____

3. Clarence bought 3 bags of beads on Monday, 5 bags on Tuesday and 4 bags on Wednesday. Each bag contains 48 beads. How many beads did he buy?

_____

4. Mardy has 25 bags of beads. She gives 6 bags to her friend, Elly, and 8 bags to her other friend, Sarah. If each bag holds 36 beads, how many beads did she give away?

_____

5. Ned has 4 large bags of beads and 7 small bags of beads. The large bags hold 25 beads and the small bags hold 15 beads. He uses 24 beads to make a necklace and 18 beads to make two bracelets. How many beads does he have now?

_____

6. Zena has 7 bags of beads, with 30 beads in each bag. She used 36 beads to make 2 necklaces and 54 beads to make 6 bracelets. She wants to make the same set of jewellery tomorrow. Does she have enough beads?

_____

SAMPLE - NOT FOR SALE

LIBRARY

The library has books, magazines and CD-ROMs. It has a total stock of 9472 books, 315 magazines and 143 CD-ROMs.

1. In the library, each set of shelves holds 100 books. How many sets of shelves are needed to hold all of the books?

_____

2. The cleaner has dusted 48 sets of shelves. How many books have been dusted?

_____

3. (a) At the start of the day, the library's computer showed there were 6841 books currently in the library. During the morning, 275 books were taken out and 97 books returned and during the afternoon, 166 books were taken out and 134 returned. How many books are now in the library?

_____

(b) Later in the day, six boxes of books, two boxes of magazines and four boxes of CD-ROMs were delivered to the library. If each box of books holds 65 books, how many books will the library now have?

_____

4. Every student in Year Four borrowed a book from the library. If there are 4 classes with 26 students in each class, how many books were borrowed?

_____

5. During stocktaking, 284 old and damaged books were removed from the shelves to be packed into boxes. If each box can hold 10 books, how many boxes are needed to pack all of the books?

_____

SAMPLE - NOT FOR SALE

AT THE STATION

1. There were 604 people on the train. At the first station, 58 people got on and 129 people got off. At the next station, 143 people got on and 72 people got off. How many people are now on the train?

   _____

2. The train has 12 carriages. Each carriage has 47 seats and can hold 65 people.

   (a) How many people can sit on the train? _____

   (b) How many people can travel on the train? _____

3. There are 36 people in the first carriage, 27 people in the second carriage and 46 people in the third carriage. Each carriage is able to hold 60 people. How many more people are needed to fill the train to capacity?

   _____

4. There are 9 carriages and each carriage is able to hold 65 people. After the stopping at the fifth station there are 329 people on the train. How many more people can board the train before it is full?

   _____

5. The train has 8 carriages and each carriage has 47 seats. There are 411 people on the train. If every seat has one person on it, how many people are not sitting down?

   _____

6. At the first station, 63 people got on and 138 people got off. At the next station, 94 people got on and 86 people got off. There are now 261 people on the train. How many people were on the train to begin with?

   _____

SAMPLE - NOT FOR SALE

Problem-solving objective
To read, interpret and analyse information.

Materials
calculator, number expander

Focus
Students explore concepts of place value and number sense. The relationships among numbers and place value are analysed and students are encouraged to find suitable numbers and disregard numbers that are not possible. Place value and number sense are needed rather than division.

New Zealand Curriculum links

Number and Algebra
Number strategies
• Use simple additive strategies with whole numbers
Geometry and Measurement
Measurement
• Create and use appropriate units to measure length (perimeter)

Discussion
Page 47

Students need to read and interpret information and use it in the context of place value. As the book has 10 equal-length chapters, an understanding of place value can be used to solve each problem. Given that the book has 690 pages we know that this number has a place value of 69 tens, so each chapter must be 69 pages long. This information can be used to calculate the various problems.

Before starting, students may find it helpful to draw up a table outlining the starting page of each chapter. We know that each chapter has 69 pages and that Chapter 1 starts on page 1, so Chapter 2 must start on page 70 and so on.

The page number that Walter has read to is given at the beginning and this information is needed to answer some of the problems. For example, in order to determine about how many pages Walter has read past the middle of the book and needs read to finish the book, it is necessary to keep in mind that there are 690 pages in the book and he has read up to page 437.

Possible difficulties
- Not considering place value to solve the questions
- Not taking into consideration the starting page of each chapter

Extension
- Change the criteria involving the number of pages in the book and the page number read to and explore the problems again based on the new criteria.

SAMPLE - NOT FOR SALE

BOOKWORMS

Walter's book starts on page 1 and has 690 pages. There are 10 equal-length chapters in the book and he has read up to page 437.

1. How many pages are in each chapter?

.....

2. Walter's favourite page is 409. Is it in Chapter 7?

.....

3. Walter reads 10 pages each morning and 10 pages each night. How long has he been reading this book?

.....

4. How many pages past the middle of the book has he read?

.....

5. Walter's favourite chapter is Chapter 3. From starting page to finish, what pages are in Chapter 3?

.....

6. The most exciting part was from pages 351 to 379. What chapter are they in?

.....

7. How many more pages does he have to read to finish the book?

.....

8. Walter's sister is also reading this book. She has read eight pages of Chapter 6. What page is she up to?

.....

9. How many more pages does she need to read to finish the book?

.....

10. If she reads 10 pages a day, how long will it take her to finish the book?

.....

SAMPLE - NOT FOR SALE

Problem-solving

To use spatial visualisation, logical reasoning and measurement to solve problems.

Materials

paper to fold

Focus

These pages explore different ways of visualising problems and analysing the possibilities that make up the whole solution. Logical reasoning is required, as well as an understanding of measurement (kilometres, metres and centimetres). In each situation, diagrams can be used to organise, sort and explore the data.

New Zealand Curriculum links

Geometry and Measurement
Measurement
• Create and use appropriate units to measure weight, length and area

Discussion

Page 49

In these problems, students need to visualise the paths that the farmer and dog take as they travel around the outside of each shape. Some of the lengths around the paddock and garden need to be determined from the diagrams and an ability to rename from kilometres to metres or centimetres to metres is required.

In the first problem, the farmer rides around the paddock more than two times, passing by corners B, C and D three times. The distance around the paddock is 5300 m which can be used, along with the distance from A to D, to calculate the distance travelled to return to D. For Problem 2, interpreting the diagram to determine the lengths is more complex, although some students may realise that finding the distance all around three times and then subtracting the distance from G to A is simpler.

Page 50

The problems on this page are essentially solved the same way; however, the provision of the image of a balance for the first problem makes it is easier to see how the mango from the top picture can be substituted using the scales in the second picture. Problem 2 requires more complex thinking.

One approach is to create a table of possible values using a process of 'try and adjust':

6 rolls and 1 loaf costs $4.20					
roll	6 rolls	loaf	4 rolls	2 loaves	Total
0.50	3.00	1.20	2.00	2.40	5.40 – too little
0.40	2.40	1.80	1.60	3.60	5.20 – too little
0.35	2.10	2.10	1.40	4.20	5.60 – too much
0.30	1.80	2.40	1.20	4.80	6.00
A loaf of bread costs $1.80. A bread roll costs 30 c.					

Page 51

This page extends the thinking about perimeters explored on page 39. With Problem 1, when the paper is folded four squares will result. Since the length of each side of the small square is 3 cm, each will have an area of 9 cm^2, so the area of the original square must be 36 cm^2. Alternatively, the large square has sides of 6 cm in length, giving an area of 36 cm^2. The second problem requires students to visualise the relationships among the squares and determine the area after the length of its sides is known.

Possible difficulties

- Unable to rename centimetres to metres or metres to kilometres
- Not using the given data to determine the sides whose length is not stated
- Unable to see how the information in the problems can balance
- Unsure of the area of a square and confusing it with perimeter

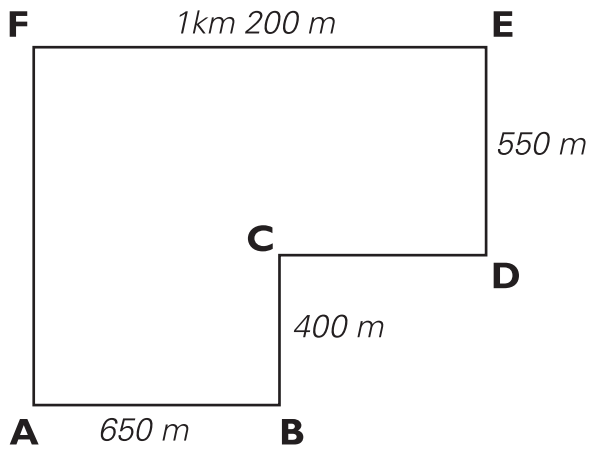
Extension

- Students could write their own problems involving distance around an irregular shape where some of the lengths have to be worked out from the information in the diagrams. Other problems could involve items on a balance or areas cut from inside an arrangement of shapes. The problems are then given to other students to solve.

SAMPLE - NOT FOR SALE

FARM TRAILS

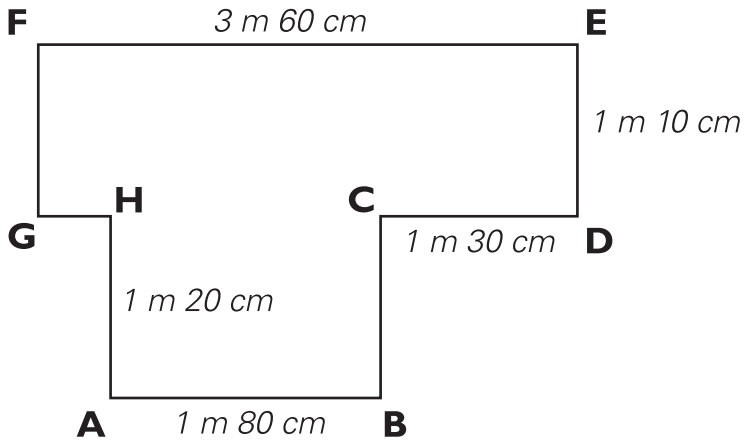
1. A farmer started at corner A of the paddock and rode his trail bike around the perimeter to see if there were any gaps in the fence. He found one hole at D and continued all the way around to A, where he picked up his tools to fix the fence. He then rode to D and fixed the hole.



To be sure the fence was completely free of holes, he rode all the way around once more until he arrived at D again.

How far did he ride to check and fix the fence? _____

2. When he got back to the farmhouse, he picked some vegetables from the garden. His dog, Blue, wasn't allowed in the garden, so he walked around the outside fence, starting at A. He ran around and around counterclockwise, only stopping when he reached G for the third time.



How far did Blue walk? _____

SAMPLE - NOT FOR SALE