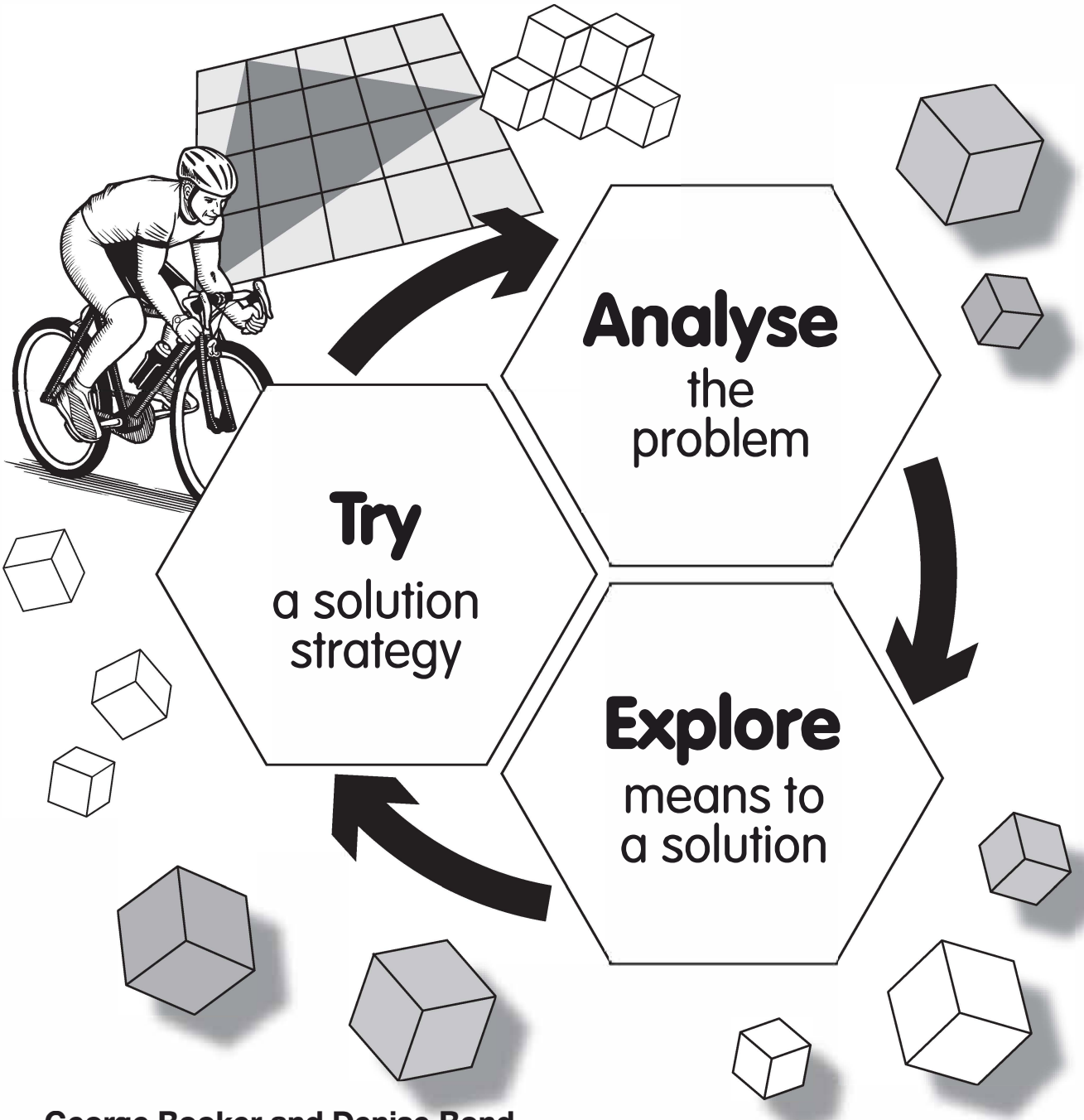
PRIMARY

BOOK F
(Ages 10–11)

CURRICULUM LINKED

Problem-solving in mathematics



George Booker and Denise Bond

SAMPLE - NOT FOR SALE

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Problem-solving

To use spatial visualisation and logical reasoning to solve problems

Materials

cubes in 4 different colours, grid paper to draw views of the shapes and nets of cubes, digital camera to photograph the shapes

Focus

These pages explore arrangements of three-dimensional shapes in order to determine how particular outcomes are formed and to investigate the patterns they exhibit. Spatial and logical thinking and organisation are required as students investigate all likely arrangements to ensure the final forms match the given criteria or visualise a given shape in terms of its component parts.

New Zealand Curriculum links

Geometry and Measurement
Shape
• Classify two and three dimensional shapes, with justification

Discussion

Page 3

This problem requires students to visualise how coloured cubes are arranged to form larger cubes. Students need to bear in mind that a cube with a height and width of 2 cubes will have a total of 8 cubes, a 3-by-3 cube will have 27 cubes, and so on—multiplying side x side x side (or side^3) to find the total. Students can then determine the difference in amount of cubes used; e.g. from a single cube to a 2-by-2 cube: 2 x 2 x 2 (or 2^3) – 1 = 7 cubes. The next difference is 27 – 8 = 19, and so on. Other students will see this pattern in terms of the amounts of cubes that are added to build the next cube. Both approaches are satisfactory and will assist in helping students see a pattern in terms of how many cubes each subsequent amount increases by; they increase by sequential multiples of six.

Page 4

Students are asked extend an arrangements of cubes in a Latin square (where each row and column has exactly one of each of 3 or more different coloured cubes), to a cube in which each horizontal and vertical row or column has exactly one cube of each colour. Students will need to visualise these patterns to guide their building of the complete cubes and some degree of 'try and adjust' will probably be needed. Encourage students to create systematic arrangements as they complete the tasks and describe the pattern of the blocks in the middle of the arrangements. This may lead some students to start with the 3-by-3 cube and build layers all around to complete a 4-by-4 cube. All arrangements would be possible with a single cube at the centre of an arrangement of an odd number of colours, while arrangements made of an even amount of colours would have a 2-by-2 cube in their centre. The actual descriptions given will vary according to the arrangement chosen for the initial Latin square, but all are essentially the same with different colours occupying different positions.

Page 5

This set of investigations encourages students to see three-dimensional shapes in terms of the component two-dimensional forms. Viewing from above, the front and one side is sufficient to capture the details of a three-dimensional shape. Another way of providing a plan for a building is to use a grid that tracks the number of cubes in each position of the base. After they make a shape of their own and construct a grid plan for another student to follow, they may be able to take digital photos from the side and front to compare with the original.

Possible difficulties

- Unable to visualise the shapes from the two-dimensional representations
- Unable to keep track of the possibilities in any of the investigations

Extension

- Investigate the pattern formed by the number of small cubes faces on the outside of the growing cubes: 6, 54 …
- Build a shape from cubes then photograph or draw the front, side and top views. Ask another student to draw a grid plan from the pictures. Make the shape from the plan and compare with the original.

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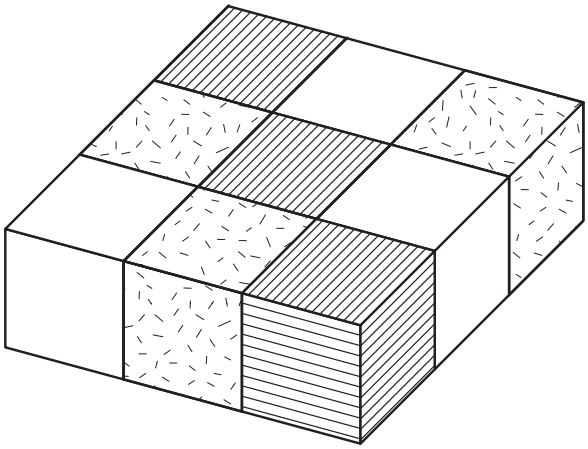
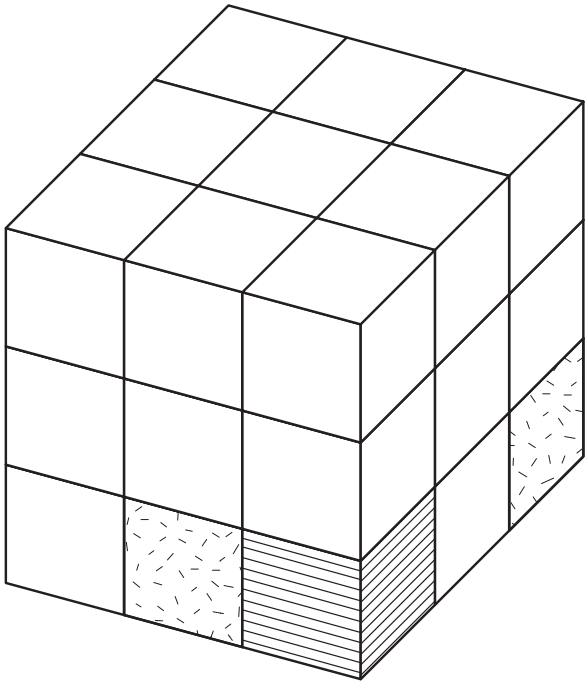
COLOURED CUBES



Use cubes in three different colours to make a square in which each row and column has exactly one cube of each colour.



Can you build this to be a cube so that on every face each row and column has exactly one cube of each colour?

1. How many cubes of each colour did you use? _____

2. (a) What coloured cube would be at the centre of your larger cube? _____

   (b) Can it be any other colour? _____

See if you can make an even larger cube from cubes in four different colours and still keep the same pattern.

3. How many of each coloured cube do you need? _____

4. What would the colour of the cube be at the centre of the larger cube? _____

   _____

5. Is it possible to continue forming larger cubes in this way? _____

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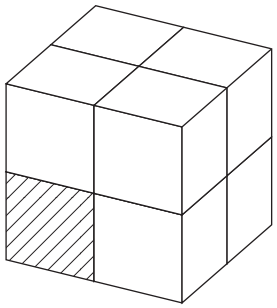
GROWING CUBES

Place a single cube in front of you. Imagine you add cubes of a second colour to it to make a larger cube.



1. How many extra cubes would you need? _____

Now imagine that you add cubes of a third colour to make a larger cube again.

2. How many cubes would you need to make a bigger cube? _____

3. How many cubes of a 4th colour would you need to add to make an even larger cube?_____

Get as many of the coloured cubes as you think you need to increase the size. See if you can make the larger cube as you imagined.

4. Explain to a friend how you worked out the number of cubes you needed to increase the size. Did you both do this the same way?

_____

_____

_____

_____

_____

5. List the number of cubes that you would need to add if you kept growing cubes in this way.

1 + 7 + _____ + _____ + _____ + _____ + _____

6. Can you describe a pattern for the way the cubes increase in size?

_____

_____

_____

_____

_____

_____

_____

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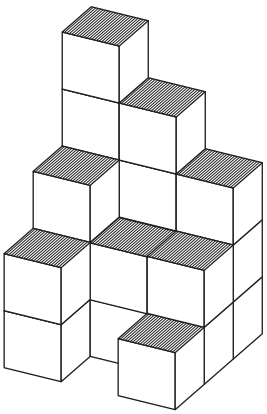
VIEWING CUBES

1. Draw what this shape would look like from the:



(a) front. (b) side.

Use some cubes to make the shape.

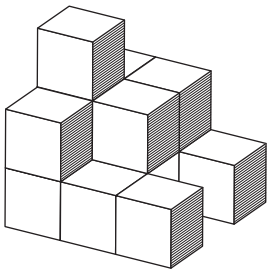
Here is a plan of this shape, using squares to show where the cubes are and numbers to show how many cubes are used.

5	4	3
3	2	2
2		1

2. Does it match the shape you made? _____

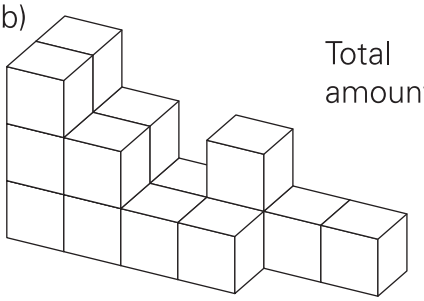
3. Write the number of cubes you would need to make each of these shapes. Get the cubes, make the shapes and draw a plan for each one.

(a)



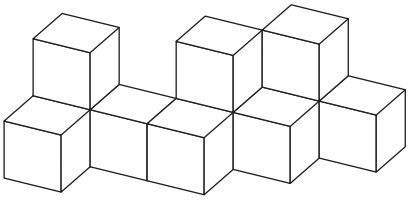
Total amount = _____

(b)



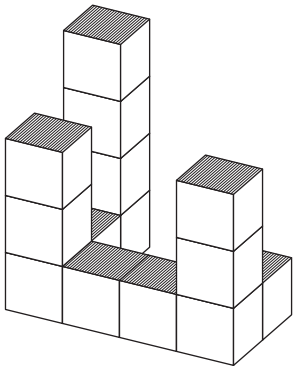
Total amount = _____

(c)



Total amount = _____

(d)



Total amount = _____

4. Make a shape of your own and then make a plan for it. Ask a friend to construct the shape from your plan and compare.

SAMPLE - NOT FOR SALE

Problem-solving

To use logical reasoning and number sense to solve problems

Materials

counters, Base 10 materials

Focus

This page explores problems based on a conceptual understanding of whole numbers and fractions and an understanding of logical reasoning to take into account the problem's context. Backtracking from the final position will assist in understanding and solving the problems but counters could also be used to keep track of what is happening. Using a diagram or calculator are other ways to sort through the information while keeping the intent of the problem in mind.

New Zealand Curriculum links

Number and Algebra
Number strategies
• Use additive strategies and simple multiplicative strategies with whole numbers
Equations and expressions
• Communicate and interpret simple addition and subtraction strategies using words, pictures and numbers • Represent objects with drawings and models

Discussion

Page 7

These problems highlight the need to carefully analyse the problem before starting on a solution. For the first problem, Lindsay is clearly not actually going to sell half an egg! At each step, there must be an odd number of eggs. When she sells these eggs, they will both get half of the nearest even number and the customer will get 1 extra egg. This means that when Lindsay had 9 eggs, the last customer must have taken 10. Logic dictates that Lindsay would have had 19 eggs to sell to her third customer. The customer before that must have taken 20 eggs when Lindsay had 39 eggs to sell. So the first customer took 40 eggs when Lindsay had 79 eggs to start with. Counters can be used to work through this thinking and a diagram can help keep track of what is needed:

Lindsay	third customer	Lindsay	second customer	Lindsay	first customer	Lindsay
9	10					
		19	20			
				39	40	
						79

A diagram can also be used for the second problem to determine the relationships among the information. Base 10 materials could also be used to model the 170 eggs and how they need to be distributed over the markets so that, at the second market, the number is 10 less than half of the first.

The third problem is similar to the first problem but this time the seller must add 2 to the number of chickens left to find half of the number he had to sell. Counters or a diagram could readily show this.

The last problem can also be solved using counters or numbers in a table. At first it seems that there must be 4 or 8 children. When these prove impossible, other ways of finding whole numbers of loaves need to be considered while bearing in mind that the number of people and the number of loaves must both equal 12.

Possible difficulties

- Unable to see how the half egg, 10 more eggs or 2 extra chickens fit into the problems
- Not thinking that a full loaf could be made up of 2 halves, 1 half and 2 quarters or 4 quarters to see how the 12 full loaves could be bought
- Simply working on the basis of calculating the numbers in the problem to obtain incorrect answers

Extension

- Ask for the number of eggs or chickens each customer bought.
- Change the numbers in the problems but leave the problem statements the same:
  – more eggs at the end of the first problem (it must be an odd number)
  – less eggs at the second market (55, 85, 115, … eggs altogether)
  – more chickens available at the end of the day or 3, 4 or 5 more each time
  – the fourth problem is a very famous problem and would be difficult to change!
- Change the problem's context but leave the numbers the same.
- Have students make up problems like these of their own and challenge others to solve them using diagrams or materials.

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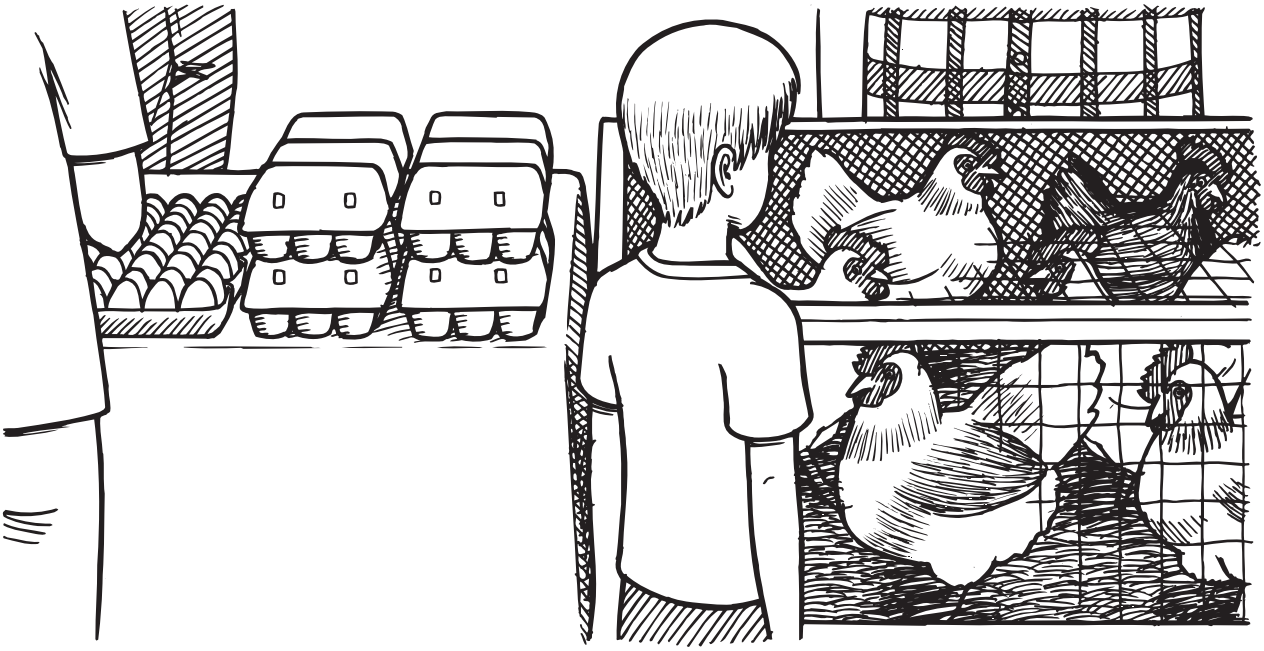
MARKET DAYS



1. My cousin, Lindsay, keeps chickens at her home and sells any spare eggs at the local farmers' market. She sold half of her eggs and another half an egg to her first customer, half the remaining eggs and another half an egg to her second customer and half the remainder and another half an egg to her third customer. At the end of the day, Lindsay noticed that she had 9 eggs left. How many eggs did she have to start with?

   _____

2. Lindsay sold 170 eggs at two different markets. She noticed that the number she sold at the second market was 10 less than half the number she sold at the first market. How many eggs did she sell at each market?

   _____

3. There was a rush to buy fresh chickens when the chicken farmer opened his market stall. The first customer bought half of the chickens he had brought to the market and two more chickens. His next customer bought half of the remaining chickens and another two chickens. The third customer bought half of what he had left and another two chickens. Sadly, the fourth customer was only able to buy one chicken. How many chickens did the farmer bring to the market to sell?

   _____

4. After the market, Lindsay joined a group of people going on a picnic. The twelve of them went first to the bakery and bought a dozen loaves of bread. Each man bought two loaves, each woman bought half a loaf, and each child a quarter of a loaf. How many men, women and children were at the picnic?

   _____

SAMPLE - NOT FOR SALE

Problem-solving
To read, interpret and analyse information

Materials
calculator, number expanders

Focus
These pages explore concepts of place value and number sense. The relationships among numbers and place value is analysed and students are encouraged to not only find possibilities but to also disregard numbers and combinations that are not possible. Place value and number sense are needed, rather than addition or multiplication.

New Zealand Curriculum links

Number and Algebra
Number strategies
• Use additive strategies and simple multiplicative strategies with whole numbers
Equations and expressions
• Communicate and interpret simple addition and subtraction strategies using words, pictures and numbers

Discussion
Page 9
This investigation requires students to read and interpret information and use this information to find a solution. As the book has 18 chapters of equal length, a calculator can be used to easily work out how many pages are in each chapter. This is information can be then readily used to calculate the last page of each chapter and subsequently the starting page of the following chapter. Before starting students may find it helpful to draw up a table outlining the starting page, the halfway point and the last page of each chapter. We know that each chapter has 124 pages and that Chapter 1 starts on page 1, so Chapter 2 must start on page 125 and so on. Entering 124 + 124 and hitting the '=' key on the calculator over and over will give you the last page of each of the 18 chapters. The page number that Nathan has read to is already provided and this information is needed to answer some of the problems; for example, in order to determine about how many pages Nathan has read past the middle of the book it is necessary to keep in mind that there are 2232 pages in the book and that he has already read up to page 1674. Similarly, this information is needed to determine how many pages need to be read to finish the book.

Page 10
These problems are very easy to investigate and solve by using a calculator, but require a deep understanding of place value to work out *why* they happen. The first problem sets out a pattern of investigation for forming two-digit numbers from any three different digits and dividing the sum of the numbers formed by the sum of the digits. The result is surprising at first, but the reason is found by considering how the digits are placed in each number. The problems then continue to include other digits, reinforcing the need to look for patterns and to understand place value.

Page 11
The puzzle scrolls contain a number of different problems, all requiring strategic thinking to find possible solutions. In most cases, students will find tables, lists and diagrams are needed to manage the data while exploring the different possibilities. In some cases, the number of possible solutions is endless and the added criteria of finding the 'smallest number' has been added overcome this dilemma. Problem 6 has at least two possible combinations to make 144.

Possible difficulties
- Difficulty in understanding place value
- Wanting to add, subtract or multiply rather than using place value or number sense
- Not using all the criteria
- Not using place value to solve the questions
- Not taking into consideration the starting page of each chapter

Extension
- Students could think up their own bookworm problems using different criteria.
- Change the criteria involving the number of pages in a book and the page number read to and solve the problems based on the new criteria.
- Students could make puzzle scroll cards where they use the existing question but change the numbers or context to make a new problem.

SAMPLE - NOT FOR SALE

BOOKWORMS

Nathan's book starts on page 1 and has 2232 pages. There are 18 chapters of equal length in the book and he has read up to page 1674.

1. How many pages are in each chapter?

.....

2. Nathan's favourite part of the story is on page 874. What chapter is it in?

.....

3. Nathan reads half a chapter each day.
   How long has he been reading the book?

.....

4. Nathan's favourite chapter is Chapter 11. What pages are in Chapter 11?

.....

5. The most exciting part was from page 1047 to page 1149. What chapters are these pages in?

.....

6. How many pages past the middle of the book has Nathan read to?

.....

7. Nathan's friend is also reading the same book. He has read 14 pages of Chapter 8. What page is he up to?

.....

8. How many more pages does Nathan's friend need to read to be where Nathan is?

.....

9. How many more pages does Nathan need to read to finish the book?

.....

10. How many more days will it take for Nathan to finish the book?

.....

SAMPLE - NOT FOR SALE

Problem-solving
To analyse and use information in word problems

Materials
calculator

Focus
These pages explore word problems that mostly require multiplication or division. Students need to determine what the problem is asking and in many cases carry out more than one step in order to find solutions. Analysis of the problems reveals that some questions contain additional information that is not needed. If necessary, a calculator can be used to assist with the calculation as these problems are about reading for information and determining what the problem is asking rather than computation or basic facts.

New Zealand Curriculum links

Number and Algebra
Number strategies
• Use additive strategies and simple multiplicative strategies with whole numbers
Number knowledge
• Know basic multiplication and division facts

Discussion
Page 13
In most cases each problem requires more than one step and involves multiplication. Problem 3 involves the concept of profit while the last investigations have a number of combinations and students can use various methods to solve it. One way would be to look at multiples of 4 and 9 and put them into a table to find the various combinations and then look for a pattern regarding the multiples.

Multiples of 4	Possible multiples of 9	Total
4	234	238
8	225	233
…	…	…
32	207	239

Page 14
The information provided in the beginning statement is needed to answer the subsequent questions. Using the original information as a basis, the numbers are changed to meet new criteria as mangoes and bananas are sorted and packed into tray and cartons. Care is needed as mangoes are packed according to number and bananas are packed according to weight. As with the investigations on the previous page some solutions may not necessarily be exact; for example, 5664 mangoes were packed into trays. If 6 per one hundred are rejected then there were 56 hundreds with 6 mangoes rejected and part of a hundred (64). This could be written as 'about 57 hundreds' or 'as 56 hundreds and two thirds of a hundred', giving solutions of 57 x 6 (to give 342) or 56 and two thirds x 6 to give a more accurate approximation of 340 (339.99).

Page 15
These problems require more than one step of calculations and involve a number of operations, including multiplication and division. The wording has been kept simple to assist with the problem-solving process. Some problems have added information (often involving weight) which is not needed to find a solution and some problems may not have an 'exact answer'; for example, Problem 2 results in 53 boxes of fish—if the penguins eat 3 boxes a day, then there is enough fish for 17 full days and 1 part day (two boxes). Students could discuss their solution in terms of 17 days and two-thirds of a day. Students should be encouraged to explore and try different ways of arriving at a solution.

Possible difficulties
- Confusion over the need to carry out more than one step to arrive at a solution
- Using all the numbers listed in the problems rather than just the numbers needed
- Difficulty with the concept of profit

Extension
- Discuss how problems can have more than one answer depending on different interpretations.
- Students could write their own problems and give them to other students to solve.
- Explore how many of these problems could be solved using the repeated addition technique on the calculator; i.e. enter '273 + 273' and then hit the '=' key—keep hitting the key and you will keep adding 273 without having to keep entering the number.

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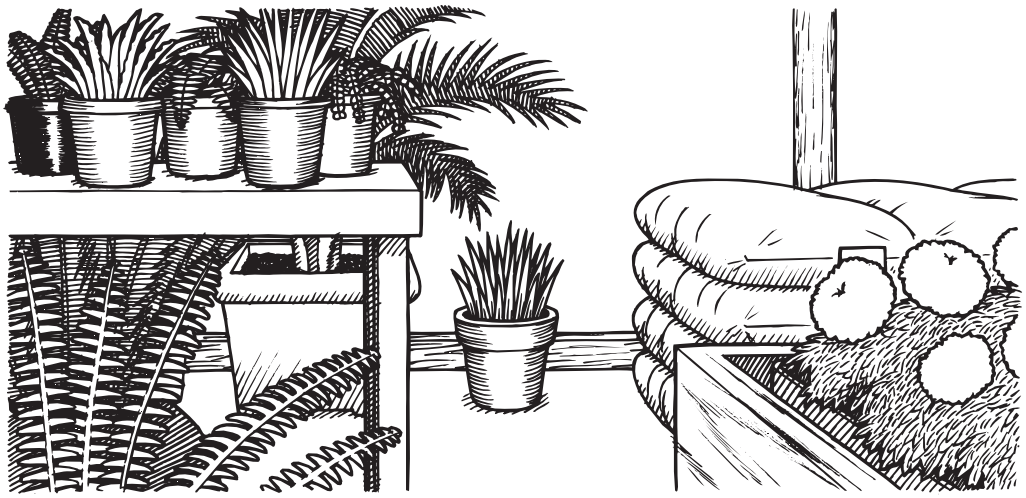
THE SEEDLING NURSERY



1. During the week, the nursery planted 258 flower punnets and 87 fern punnets. A flower punnet holds 8 seedlings and a fern punnet holds 6 seedlings. How many seedlings were planted during the week?

_____

2. During the morning, 134 flower punnets were fertilised and 120 fern punnets were watered. During the afternoon, another 56 punnets were watered and 48 punnets were fertilised. If there are 6 seedlings in each punnet, how many seedlings were watered?

_____

3. (a) The nursery sold 134 bags of bark chips over 7 days. If each bag sells for $29, how much money did the nursery receive from selling bark chips?

_____

   (b) If each bag of bark chips costs the nursery $11 to produce, how much profit did the nursery make from the bark chips?

_____

4. During the morning, the nursery used punnets which held 4, 6 and 8 seedlings. How many seedlings were planted if 115 small punnets were used?

_____

5. During the afternoon the nursery used punnets that held either 4 or 9 seedlings. If 239 seedlings were planted, what combination of punnets could have been used?

_____

6. The next day, the nursery used punnets that held either 6 or 8 seedlings. If 376 seedlings were planted, what combination of punnets could have been used?

_____

SAMPLE - NOT FOR SALE

THE TROPICAL FRUIT ORCHARD

The tropical fruit farm has 14 hectares of mangoes and 6 hectares of Cavendish bananas.

1. Each hectare of mangoes has 94 trees and is irrigated on a rotation basis. Two and a half hectares are watered each day before sunrise. How many trees are watered each day and how many days would it take for all of the trees to be watered?

_____

2. All of the bananas are irrigated over a 3 day period. If each hectare has 1450 banana plants, how many hectares are watered each day?

_____

3. (a) In the packing shed, mangoes are sorted and placed into trays that hold 16 mangoes. Meanwhile, bananas are sorted into 13 kg cartons. When the truck arrives to transport the fruit to the market, there are 198 cartons and 354 trays ready to be transported. How many mangoes are packed and ready to go?

_____

(b) Some damaged and blemished mangoes are rejected and are not packed into trays. About 6 mangoes per 100 are rejected. Approximately how many mangoes were rejected during the day?

_____

(c) How many kilograms of bananas were packed during the day?

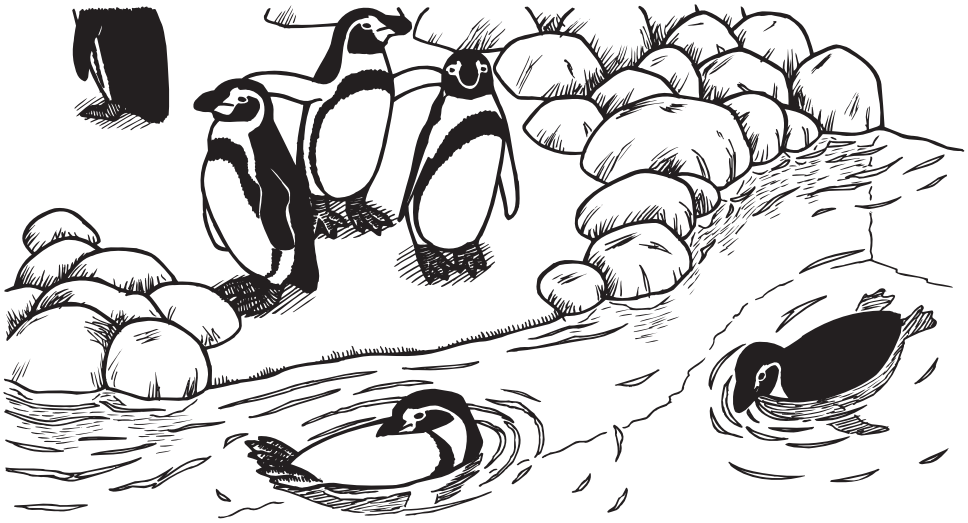
_____

4. After 3 years, new banana plants are needed. Each year some plants are removed and replaced with new plants so that each plant is replaced over a three-year cycle. How many new plants are replaced each year?

_____

SAMPLE - NOT FOR SALE

ANIMAL SAFARI PARK



1. At Animal Safari Park, the tropical birds in the walk-through aviary eat 7.5 sacks of seed a week. Each sacks weighs 40 kg and costs $63. How many sacks are used over one year?

_____

2 The penguins eat 3 boxes of small fish a day. Each box weighs 2.4 kg and costs $6.50. The park has just purchased $344.50 worth of fish. How long will this last the penguins?

_____

3. The elephants eat 7 bales of hay over two days. Each bale of hay costs $12. How much would it cost to feed the elephants during the month of July?

_____

4. Seals eat 5 cartons of medium-sized fish a month. Each carton weighs 12 kg, contains 6 boxes of fish and costs $57. How many kilograms of fish do the seals eat over a year?

_____

5. The carpark has 12 sections for cars to park in. Each section holds up to 176 cars. If sections 1–6 are completely full and sections 7–9 half full, how many cars are in the parking lot?

_____

6. The area to see the birds of prey display show has 23 rows of seats, with 27 seats in each row. If it is two-thirds full, how many more people can watch the show?

_____

7. The cafeteria seats 250 people at tables inside and 65 people outside. Currently, there 23 vacant seats inside and 16 seats outside. How many people are seated?

_____

SAMPLE - NOT FOR SALE